



α_S from Lattice QCD: progresses and perspectives for a realistic full-QCD determination of the running Strong coupling

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Some very recent computations of $\alpha_{\overline{\text{MS}}}(M_Z)$ from $N_f = 1 + 1$ lattice simulations and of the running of the Strong coupling, obtained from the lattice ghost-gluon vertex, over a large momentum window are very briefly reviewed.

35th International Conference of High Energy Physics

July 22-28, 2010

Paris, France

*Speaker.

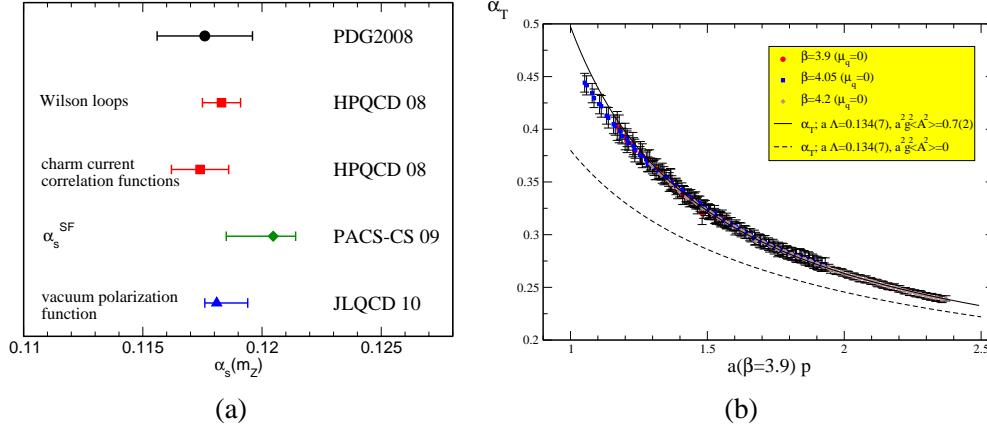


Figure 1: (a) Summary of $\alpha_{\overline{MS}}(M_Z)$ from $n_f = 2 + 1$ lattice simulations, compared with PDG 2008 average (black) [10] discussed in the text. The red points are for determinations using staggered fermions, the green for one using Wilson and the blue overlap fermions. The plot is taken from [9]. (b) α_T from the lattice, after applying the appropriate lattice-artefacts curing procedure, confronted to the continuum formula obtained from PT and including OPE non-perturbative corrections. The solid line is for the complete non-perturbative expression, while dotted stands only for the perturbative four-loop one, α_T^{pert} . The momentum in the x-axis is expressed in lattice units of $a(\beta = 3.9)^{-1}$. The plot is taken from ref. [7].

1. Introduction

$\Lambda_{\overline{MS}}$ is the scale of strong interactions. This parameter has to be taken from experiment and can be determined from the running of the QCD coupling constant. This latter had been calculated in the past by following a variety of non-perturbative ways on the lattice (see [1, 2, 3, 4, 5, 6] and references therein) from quenched and $N_f = 2$ gauge configurations; although those results cannot be properly compared with the experimental determinations of the Strong coupling, $\alpha_s(M_Z)$, because of the inaccessibility of the threshold scales of $\mu = \mu_{u,d,s}$. We will very shortly comment first on the very recent reported progress on the lattice determination of $\alpha_s(M_Z)$ from $N_f = 2 + 1$ simulations, where perturbation theory is used for the matching at the threshold for the charm mass, $\mu = O(1)$ GeV, from $N_f = 3$ to $N_f = 4$, implying not to take into account the non-perturbative effects, still important at this scale. Then, we will focussed on the study of the running itself of the Strong coupling through the comparison between the perturbative and lattice determinations of α_s from the ghost-gluon coupling over a large momentum window [7]. This has been done from quenched lattice simulations and with $N_f = 2$ twisted mass quark flavours [8] and reveals the presence of a dimension-two $\langle A^2 \rangle$ condensate, signaling that momenta considered in lattice simulation are in a non-perturbative region.

2. $\alpha_{\overline{MS}}(M_Z)$ from the lattice

There have been very recent estimates of $\alpha_{\overline{MS}}(M_Z)$ by applying different procedures (for a recent report, see section 4.1 of [9]) from lattice simulations.

In ref. [11], the coupling α_V defined from Wilson loops through lattice perturbation theory is computed and then matched to $\alpha_{\overline{MS}}(\mu)$ at three-loop. The authors of ref. [12] use the continuum

three-loop expression of a moment of charm current-current correlation function and get $\alpha_{\overline{\text{MS}}}$ by comparison with the lattice estimates of the same moment. The work of ref. [13] use step scaling of the SF coupling, where the renormalization scale is set from the inverse linear lattice extension $\mu = 1/L$, and matching to $\overline{\text{MS}}$ with three-loops PT. Finally, in ref. [14], the continuum vacuum polarization function has been obtained through operator product expansion and the relevant coefficients has been calculated up to three-loops, the renormalization scale being set from the size of the injected momentum at the current. The results of $\alpha_{\overline{\text{MS}}}(M_Z)$ estimates from those procedures appear summarized in Fig. 1.(a).

3. Lattice computation of the coupling in the Taylor scheme

In ref. [7], we calculate the strong coupling constant from the ghost-gluon vertex through

$$\alpha_T(\mu^2) \equiv \frac{g_T^2(\mu^2)}{4\pi} = \lim_{\Lambda \rightarrow \infty} \frac{g_0^2(\Lambda^2)}{4\pi} G(\mu^2, \Lambda^2) F^2(\mu^2, \Lambda^2), \quad (3.1)$$

where F and G are the ghost and gluon dressing functions and $\Lambda = a^{-1}(\beta)$ is the regularisation cut-off. This coupling is renormalized in the MOM Taylor scheme, where the ghost-gluon vertex is finite and the only form factor surviving goes to 1 [15] because the incoming ghost momentum is taken to vanish. Here g_0 is the bare strong coupling and μ the renormalization scale. This definition can be used in a lattice determination and is to be compared with a theoretical formula in order to extract Λ_{QCD} :

$$\alpha_T(\mu^2) = \alpha_T^{\text{pert}}(\mu^2) \left(1 + \frac{9}{\mu^2} \frac{g_T^2(q_0^2) \langle A^2 \rangle_{R, q_0^2}}{4(N_C^2 - 1)} \right), \quad (3.2)$$

where $\alpha_T^{\text{pert}}(\mu^2)$ is available at the four-loop level [7, 16] in PT and, to cure the observed mismatch between lattice and perturbative determination, a non-perturbative OPE correction to the perturbative formula is to be considered. This accounts for the minimal power correction associated to the presence of a dimension-two $\langle A^2 \rangle$ condensate [5, 7]. The Λ_T in the MOM Taylor-scheme and the dimension-two gluon condensate are to be obtained from the confrontation, over a large momentum window, of eq. (3.2) to the lattice data computed from eq. (3.1) and properly cured of lattice artefacts, as explained in [5]. Then, we applied this procedure and exploited the ETMC lattice configurations [17] with $N_f = 2$ twisted-mass dynamical quark flavours and, after the conversion of Λ_T to $\overline{\text{MS}}$, obtain (see Fig. 1.b):

$$\Lambda_{\overline{\text{MS}}} = (330 \pm 23) \times \frac{0.0801 \text{ fm}}{a(3.9)} \text{ MeV}, \quad g^2(q_0^2) \langle A^2 \rangle_{q_0} = (2.4 \pm 0.8) \times \left(\frac{0.0801 \text{ fm}}{a(3.9)} \right)^2 \text{ GeV}^2; \quad (3.3)$$

where $a(3.9) = 0.0801(14) \text{ fm}$ [17]. Of course, with only two sea quark flavours, the computation of $\alpha_{\overline{\text{MS}}}(m_Z)$ is still inaccessible. A computation from $N_f = 4$ lattice simulations is now in progress.

4. Conclusions

We shortly reported on some very recent computation of $\alpha_{\overline{\text{MS}}}(m_Z)$ from $N_f = 2 + 1$ lattice simulations, allowing a matching from $N_f = 3$ to $N_f = 4$ wich uses PT and neglects the impact of the

still important non-perturbative impact at the charm quark mass. Then, we also reported on the computation of the Strong coupling from the ghost-gluon vertex over a large momentum window, which reveals the impact of the non-perturbative effects at energies of the order $O(1)$ GeV and leads to an estimate of $\Lambda_{\overline{\text{MS}}}$ for $N_f = 2$ consistent with other independent computations.

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